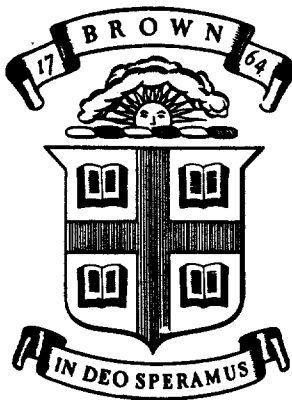


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## APPLIED MATHEMATICS

Letitz contract - The mechanical behavior of  
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rates

Technical Report No. 9

A Method for Determining Material Properties  
at High Rates of Shearing Strain

by

J. W. Phillips

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A Method for Determining Material Properties  
at High Rates of Shearing Strain

by

J. W. Phillips

Abstract

The details of an experimental program suitable for determining the mechanical properties of certain materials at high rates of shearing strain are given. The major contribution of this work is a description of a successful method for producing sharp torsional pulses in the main loading rod of a torsional split-Hopkinson bar, by means of the simultaneous detonation of two explosive charges at the ends of long "pre-load" bars in contact with the main loading rod. The method could be used for the study of the linear viscoelastic properties of materials like high polymers, and could also be employed in the study of the non-linear behavior of materials at high rates of shearing strain.



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## I. INTRODUCTION

An understanding of the mechanical behavior of materials is essential in many fields of engineering science today. In particular, there is an increasing need for basic data concerning structural materials which are subjected to rapid loading, and several experimental techniques have been developed for collecting such data.

A conventional crosshead-type testing machine can be used for dynamic testing if the required strain rates are not too large, or alternatively, if the required loading times are not too small (say less than 1/10 second). For loading times in the millisecond range (1 millisecond =  $10^{-3}$  second), such large-scale testing machines become ineffective due to their inherent inertia, and resort is made to other types of testing device.

For example, in the study of viscoelastic materials, the "lumped inertia" mechanical systems like torsional and longitudinal pendula have been effective [1]. Other methods producing millisecond loading times involve impact; an example is the ball-bouncing experiment, which has been used for studying metals [2] and hard plastics [3].

Experimental procedures involving loading times considerably smaller than 1 millisecond inherently depend upon wave propagation within the loading device and/or specimen, since it is on the microsecond ( $10^{-6}$  second) scale that stress waves propagate through distances on the order of one inch in most

structural materials (see Kolsky's book [4], p. 201). Indeed, one-dimensional wave propagation in cylindrical rods forms the underlying theory for the so-called split Hopkinson pressure bar, which was introduced by Kolsky [5] and subsequently used and modified by him [6] and others (for example, Lindholm and Yeakley [7]) for use in studying the rapid, elastic-plastic deformation of metals in compression. Lindholm and Yeakley [8] have recently extended the basic method to tensile loading as well.

The basic compressional split-Hopkinson bar experiment consists of sandwiching a thin disk of the specimen material between the ends of two lengths of round stock in such a way that all the centerlines are collinear. A pressure pulse, such as that produced by impact or by explosion, is applied directly to the free end of the first (or "loading") rod. This pulse propagates along the loading rod and eventually meets the specimen, at which point part of the pulse is reflected back into the loading rod, and part of the pulse is transmitted into the second (or "transmission") rod. It is essential that both the loading rod and the transmission rod remain elastic during the experiment, so that the recorded time variations of surface strain or displacement in these rods can be interpreted with the aid of the elastic bar wave theory. The deformation of the specimen, on the other hand, is in general not elastic; the important assumption with regard to the specimen is that the stress distribution within it is fairly uniform along its length at any given instant, even though the stress intensity varies rapidly with time as the

pulse propagates through it. Details of the data reduction involved appear in [5,8]. It is found [5] that the time integral of the difference between the incident and transmitted stress pulses, which (if the above assumptions hold) is equal to the time integral of the reflected stress pulse [8], is proportional to the average strain in the specimen; also, the transmitted stress pulse is proportional to the average stress in the specimen. Thus with two strain gages, one mounted on the loading rod (far enough away from the specimen so that the incident and reflected pulses do not overlap in time) and one mounted on the transmission rod, it is possible to determine independently the average strain and average stress in the specimen as functions of time. These functions can be cross-plotted to yield a stress-strain curve with time as a parameter.

The assumption regarding the uniformity of stress along the specimen has not gone unchallenged [9]. Objections involving the radial inertia [5] of the specimen, and the "barreling" of the specimen due to end constraints and/or friction at the interfaces have been raised and examined by several authors, as Baker and Yew [10] point out.

A modified split Hopkinson bar which avoids these radial inertia and barreling problems was introduced by Baker and Yew [10] and subsequently used by them and their colleagues [11,12] to investigate strain-rate effects in a number of metals. Torsional stress pulses, as opposed to compressional ones, were employed, and the loading and transmission "bars", as well as the specimen,



were actually thin-walled tubes. The incident torsion pulse was produced by pre-twisting a portion of the loading tube and by mechanically releasing the clamp which isolated this pre-twisted portion from the rest of the system. A torsional pulse with a rise time as short as 30 microseconds then propagated toward the specimen in a manner directly analogous to the compressional split Hopkinson bar test.

Pope, Vreeland, and Wood [13] have introduced an ingenious torsion pulse generator which uses, in place of a clamp, a piece of aluminum foil which is evaporated by electrical discharge. Essentially square pulses with rise times of about 5 microseconds are shown to be obtainable by this method.

It is the purpose of this report to outline an alternative method for producing torsional pulses suitable for a split Hopkinson bar test. In contrast to the previously mentioned method of releasing stored torque [10-13], the present method uses a pair of explosive charges which are fired simultaneously, as described in the following section on Experimental Procedure. It is demonstrated later (see Results) that a torsional pulse whose total duration is about 30 microseconds can be produced by the present technique. Further, it appears that the principal application of the method will be the study of the torsional behavior of materials like plastics, such as those previously tested in compression [5].

## II. EXPERIMENTAL PROCEDURE

Summary. The complete torsional split Hopkinson bar developed in the present study is shown in Fig. 1, and is detailed in Fig. 2. The method calls for firing simultaneously the two lead azide charges located at the ends of two parallel rods called pre-load bars. The other ends of these pre-load bars are cemented into milled cavities located near the end of the main loading rod. The centerlines of the pre-load bars do not coincide, so that the short longitudinal pulses produced in them during firing give rise to a single torsional pulse in the main loading rod upon their arrival. This torsional pulse then propagates toward the specimen and is monitored by semiconductor strain gage rosettes with mutually perpendicular elements both before and after it meets the specimen. An oscilloscope, triggered by a delay unit which senses the explosion electrically, records the outputs of both strain gage rosettes as functions of time.

Description of charges. Each charge consists of a small amount of lead azide ( $\text{PbN}_6$ ), which is a white, powdery explosive, normally stable under ambient conditions but easily detonated by intense heat. The lead azide is contained in a polyethylene capsule whose dimensions are indicated in Fig. 3. The "fuse" is a short (active) length of fine tungsten wire through which an electric current can be passed by means of ordinary hookup wire connected ultimately to the capacitor discharge system (described below). The charge is capped with double-sticky tape and is joined

to one face of a short aluminum pellet. The other face of this pellet is ground flat for the purpose of mating with the ground end of the pre-load bar. A greased interface between the pellet and the pre-load bar suffices for the transmission of the longitudinal pulse, and at the same time allows the pellet (which is plastically deformed during a test) to be discarded later.

It is essential that both charges detonate simultaneously (i.e. within a fraction of a microsecond of one another), so that the longitudinal pulses in the preload bars yield only pulses of torsion, and not a combination of torsion and flexure, in the main loading rod.

Capacitor discharge system. To insure this simultaneity, it is not sufficient to heat the tungsten "fuses" slowly. They must heat up, vaporize, and form a conducting plasma within a small fraction of a microsecond, i.e. they must explode. Carlsson and Gilman [14] give the details for an elaborate capacitor discharge system suitable for exploding two (or more) tungsten wires in parallel. The method used here is a simplified version of theirs, in that the spark-inducing trigger circuit has been eliminated, and the spark gap has been replaced by a set of normally open contacts which are closed for firing. See Fig. 4. When the 0.5 microfarad electrolytic capacitor (charged usually to 2000 volts) is discharged, a triggering signal of about 5 volts (peak) is produced in the short length of hookup wire which is in series with the parallel-connected exploding wires.

Other equipment. This triggering signal is fed to the variable (0-1500 microsecond) delay unit which was set, in the present experiments, for 100 microseconds. The delay is used in order to make full use of the horizontal sweep of the Tektronix 551 dual-beam oscilloscope (which has no built-in delay). A Hewlett-Packard 523 C microsecond counter is used to monitor the delay unit.

Two Kyowa type KSP-2-F3 ( $90^\circ$ , 2 element) strain gage rosettes are employed to make the dynamic torsional strain measurements. These strain gages have a gage length of 2 millimeters, a nominal gage resistance of 130 ohms, and a nominal gage factor of 126. They are activated by ordinary dynamic circuits consisting of one battery (67 volts, nominal) and slightly adjustable ballast resistors (of about 3000 ohms). The outputs of each pair of strain gages are differenced by a Tektronix type W plug-in unit used with the 551 oscilloscope.

Timing and length considerations. Some of the length dimensions employed in the torsional split Hopkinson bar, as described above, are subject to certain minimum values imposed by the finite wave speeds involved and by the pulse durations expected. It is noted in Fig. 2 that the loading-bar strain gage rosette, for example, is located 4 inches away from the specimen. Now, in aluminum, the inverse (elastic) shear wave speed is about 8 microseconds/inch, so that it would take approximately 64 microseconds for the head of an incident torsional pulse to travel from this first strain gage to the specimen and return as the head of the reflected pulse. Consequently, incident torsional

pulses whose durations are less than 64 microseconds can be distinguished from the subsequent returning ones with the present arrangement. (It was expected that torsional pulses considerably shorter than this would be produced by the pre-load bar technique.)

Also, the pre-load bars are subject to a minimum length due to the problem of double reflection of the longitudinal pulses within them. It will be seen that, due to the inevitable partial reflection of the longitudinal pulse at the main (torsion) loading bar, a small reloading of this bar will occur at a time  $2L/C_0$  later, where  $L$  is the length of the pre-load bars and  $C_0$  is the elastic bar wave speed. For aluminum,  $1/C_0$  is approximately 5 microseconds/inch, so that, with  $L = 12$  inches (as shown in Fig. 2), this reloading would begin about 120 microseconds after the original loading. It is further seen that this is about the minimum allowable "dwell" time consistent with the torsion pulse duration and reflection criteria associated with the first measuring strain gage.

A minimum length is imposed on the transmission bar in order to avoid interference between the transmitted pulse and its own reflection from the free end, at the second monitoring station. In the present setup, this calls for a minimum of 4 inches between the second strain gage rosette and the free end of the transmission bar.

Specimen preparation and mounting. A dummy specimen has been used in the tests conducted so far. It consists of a thin (0.200 inch) wafer of aluminum cut from the same 1 inch-round

stock from which the loading and transmission bars were cut. All mating surfaces were milled flat, but were not ground or polished. Baldwin-Lima-Hamilton EPY-150 epoxy strain gage cement was used to join the specimen to the loading and transmission bars, and during the curing process the whole assembly was moderately compressed in a longitudinal jig to reduce the adhesive thicknesses as much as possible.

It is presumed that a similar process could be employed for preparing specimens of other materials of interest. Note that, with torsional waves, there is no ambiguity with regard to the desired interface condition (e.g. lubricated, non-lubricated, or cemented). They must be cemented or rigidly joined in some way; and yet, the problem of specimen "barreling" does not arise because of the equivoluminal mode of specimen deformation.

### III. RESULTS

Some tests have been conducted with the torsional split-Hopkinson bar described in the previous section; the pertinent results are given and discussed in this section. It is important to remember that, with the dummy specimen in place, the test apparatus can be "calibrated", i.e. certain basic questions regarding the ability of the pre-load bar method to produce good torsional pulses in the main loading rod can be answered. Note that when the specimen is of the same material and has the same cross-sectional properties as both the loading and transmission bars, the split-Hopkinson bar reduces to a single, isotropic, elastic, circular rod.

In such a medium, torsional waves traveling in the fundamental mode are non-dispersive [4] and consequently torsional pulses should propagate along it without change in form. Figure 6(c) shows the time variations of shearing strain measured at two points on the split-Hopkinson bar, as indicated in Fig. 2. It will be seen that the two traces are virtually identical in shape and that the second trace "lags" the first by about 67 microseconds. This is the correct time for transit of torsional waves, as can be calculated by noting the distance traveled (8.2 inches) and the inverse shear wave speed for aluminum (8.2 microseconds/inch [4, p. 201]). From these observations regarding non-dispersiveness and transit time, it can be concluded that the pre-load bar method does in fact produce torsional pulses.

The results in Fig. 6(c) do not prove that no other types of wave (e.g. longitudinal bar wave or flexural wave) were simultaneously produced in the loading rod, since the strain gage rosettes were instrumented so as to be insensitive to these types of wave even if they were present. However, earlier tests in which the presence of a longitudinal pulse could have been detected showed that its magnitude was negligible in comparison to the torsional pulse produced. As for the existence of flexural pulses, indirect evidence from the results in Fig. 6(b) demonstrates that, when two charges are used, these flexural pulses are also negligible in comparison to the torsional pulses produced. To see why this is so, note that the only difference between the tests of Fig. 6(b) and 6(c) is that one explosive charge was used in the first while two charges were used in the second. Now obviously flexural pulses are produced in the loading rod when only one charge is used, and indeed a dispersive "wake" is seen in the traces of Fig. 6(b), corresponding to the higher phase velocity components of flexural waves [4], despite the efforts to cancel out flexural waves electronically. This non-cancellation can probably be attributed to misalignment of the strain gage rosettes and to the separation of gage elements within each rosette. In any event, dispersive "wakes" are not seen when two charges are employed, and this indicates that the production of flexural pulses has been kept to a minimum by the experimental procedure outlined in Section II.



Attention is now focused on the duration of the torsional pulse produced with the pre-load bar technique. It is seen from Fig. 6(c) that the total duration of this pulse is about 30 microseconds. The question arises as to why this pulse is so long, given that the lead azide charges detonate in about 2 microseconds. Part of the answer is found in Fig. 6(a). In this figure, the top trace corresponds to the output of a single, longitudinal strain gage centrally located on a pre-load bar 18 inches long. Only one pre-load bar and one charge were used in this particular test. The lower trace of Fig. 6(a) corresponds to the output of the first torsional strain gage rosette of the split-Hopkinson bar. Note that the longitudinal pulse in the pre-load bar is about 10 microseconds long, and that it is immediately followed by a Pochhammer-Chree "tail" [15]. Thus part of the lengthening of the original pressure pulse has occurred in setting up a longitudinal pulse within the pre-load bars, and this is probably due not only to longitudinal dispersion within the rod, but also to geometric dispersion [16] and possibly to plastic deformation in the vicinity of the explosively loaded end.

The fact that the torsional pulse in the main loading bar has approximately twice the duration of the longitudinal (pre-load bar) pulse which generated it is probably due to the complicated, three-dimensional nature of the junction between the pre-load bars and the main loading bar. In other words, within the first few diameters of the main loading rod, the necessary conversion from essentially two point sources of load application

into a well-developed, linearly varying stress distribution is accompanied by a geometrical broadening of the source pulse.

Note also in the upper trace of Fig. 6(a) that part of the original longitudinal pulse in the pre-load bar is reflected back into the pre-load bar; this pulse is destined to return from the explosively-loaded end as one of tension, thus reloading the main bar in a sense opposite to that of the original loading. Indeed, the upper traces of Figs. 6(b,c) bear this prediction out: a "negative" torsional pulse is observed to be centered at the 8-centimeter graticule, which is the correct time of arrival for such a pulse (see discussion of 'timing and length considerations' in Section II).

It remains to compute the magnitude of the torsional stress pulses produced with the present method. A quasi-static calibration procedure was employed in which one end of the split-Hopkinson bar was clamped and the other end twisted by means of a lever-arm and dead weight load. The results of this calibration showed that the torsional strain gage responses were equal to 1 millivolt for each 2.86 inch-pounds of torque, and for the 1-inch solid bar used, this corresponds to a maximum shearing stress of 14.5 psi per millivolt of signal amplitude. Now, the maximum amplitude of the signals produced during the test of Fig. 6(c) was about 140 millivolts, indicating the production of shearing stress (near the surface of the aluminum bar) of about 2000 psi.

This is a fairly low value of shearing stress, in comparison to (say) the yield shearing stress of 6061 aluminum (3000 psi in the 0-condition and 20,000 psi in the T6-condition [17]). In

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addition, it is noted that the use of solid disk specimens and solid loading and transmission bars (as described in Section II) restricts the application of the present method to studies of the linear (time-dependent or time-independent) properties of materials in shear. The reason for this is that, in torsion, the shearing strain varies continuously throughout a given cross-section, and unless the specimen is assumed a priori to exhibit linearity between its stress and strain histories, it is not possible to speak of 'the' stress in the specimen, even if the shearing strain is known to vary linearly across the cross-section.

Thus the present method is suitable for the study of linear elastic and linear viscoelastic materials where sharp, small-amplitude, torsional pulses are required. In principle, the specimen may be made very short, so that high shearing strain rates are possible. Making the specimen short is also desirable from the point of view of keeping the stress distribution uniform along the specimen.

The present method could be modified for the study of the non-linear behavior of materials in shear, through the use of thin-walled, tubular specimens, for which it is permissible to speak of 'the' shearing stress, or average shearing stress, across the thickness of the tube wall. Then, to avoid mismatch due to an abrupt change in cross-sectional properties, one should also use tubular loading and transmission "bars", as Baker and Yew [10] did. Attempts were made in the present study to produce torsional pulses in thin-walled tubes by the pre-load bar

technique, and indeed some success has been achieved in this respect. It must be admitted that the apparent production of other modes of tube deformation has plagued this particular study so far, but the desire to obtain the non-linear properties of materials at high shearing rates of deformation is felt to be sufficient justification for continued research into the use of thin-walled tubes with the present pre-load bar technique. It is also felt that, with the use of thin-walled tubes, the necessary higher torsional stress levels associated with the non-linear behavior of many structural materials can be induced with the pre-load bar technique described in this report.

#### IV. CONCLUSIONS

On the basis of the foregoing results from experiments using the torsional split-Hopkinson bar, as detailed in Section II, the following conclusions can be drawn:

(1) with 2 lead azide charges and 2 long pre-load bars, it is possible to generate a torsional pulse and only a torsional pulse, within a solid aluminum rod;

(2) with the present setup the total duration of this pulse, in the main loading rod, is about 30 microseconds long, and the maximum shearing stress produced in the rod is about 2000 psi;

(3) the linear behavior of soft materials like plastics could be tested at high rates of shearing strain with the experimental setup as it now stands; and

(4) with suitable modification, the method could probably be used for the study of structural materials like aluminum, anelastically deformed at high rates of shearing strain, but difficulties associated with the effective generation of torsional pulses in thin-walled tubes with the pre-load bar technique are anticipated.

## ACKNOWLEDGMENTS

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## REFERENCES

- [1] J. P. Walsh, "An Experimental Investigation of the Dynamic Tangent Moduli of Polyethylene," Brown University Report DA-358/8 (Army Research Office, Durham)(1968).
- [2] J. M. Lifshitz and H. Kolsky, "Some Experiments on Anelastic Rebound," Journal of the Mechanics and Physics of Solids, vol. 12 (1964), p. 35.
- [3] J. W. Phillips and H. H. Calvit, "Impact of a Rigid Sphere on a Viscoelastic Plate," Journal of Applied Mechanics, vol. 34E (1967), p. 873.
- [4] H. Kolsky, Stress Waves in Solids (Clarendon Press, 1953), Also Dover reprint (1964).
- [5] H. Kolsky, "An Investigation of the Mechanical Properties of Materials at Very High Rates of Loading," Proceedings of the Physical Society, vol. 62B (1949), p. 676.
- [6] H. Kolsky and L. S. Douch, "Experimental Studies in Plastic Wave Propagation," Journal of the Mechanics and Physics of Solids, vol. 10 (1962), p. 195.
- [7] U.S. Lindholm and L. M. Yeakley, "Dynamic Deformation of Single and Polycrystalline Aluminum," Journal of the Mechanics and Physics of Solids, vol. 13 (1965), p. 41.
- [8] U.S. Lindholm and L. M. Yeakley, "High Strain-rate Testing: Tension and Compression," Experimental Mechanics, vol. 8 (1968), p. 1.
- [9] J. F. Bell, "An Experimental Diffraction Grating Study of the Quasi-static Hypothesis of the Split Hopkinson Bar Experiment," Journal of the Mechanics and Physics of Solids, vol. 14 (1966), p. 309.
- [10] W. E. Baker and C. H. Yew, "Strain-Rate Effects in the Propagation of Torsional Plastic Waves," Journal of Applied Mechanics, vol. 33 (1966), p. 917.
- [11] R. R. Nachlinger and C. H. Yew, "Behavior of Aluminum, Iron, and Lead under Torsional Impact," University of Texas Technical Report EMRL TR 1014 (1967).
- [12] C. H. Yew and H. A. Richardson, "The Strain Rate Effect and the Incremental Plastic Wave in Copper," University of Texas Technical Report EMRL TR 1015 (1967).

- [13] D. P. Pope, T. Vreeland, Jr., and D. S. Wood, "Machine for Producing Square Torsion Pulses of Microsecond Duration," The Review of Scientific Instruments, vol. 35 (1964), p. 1351.
- [14] A. J. Carlsson and J. J. Gilman, "Method for Measuring Sonic Dislocation Velocities," The Review of Scientific Instruments, vol. 34 (1963), p. 860.
- [15] D. Y. Hsieh and H. Kolsky, "An Experimental Study of Pulse Propagation in Elastic Cylinders," Proceedings of the Physical Society, vol. 61 (1958), p. 608.
- [16] H. Kolsky, "The Propagation of Longitudinal Elastic Waves along Cylindrical Bars," Philosophical Magazine, vol. 45 (1954), p. 712.
- [17] K. Angermayer, Structural Aluminum Design (Reynolds Metals Company, 1962).



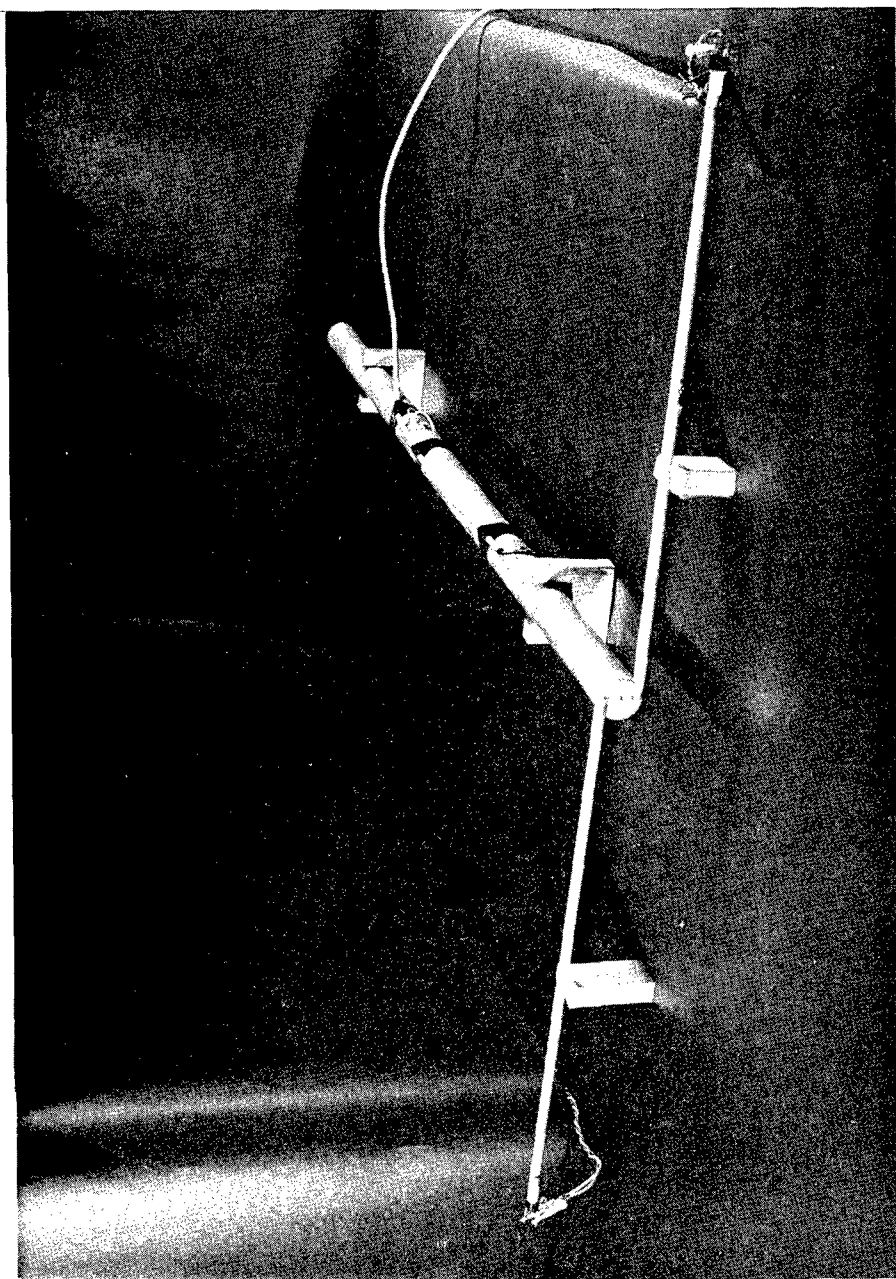


Fig. 1.—Photograph of the torsional split Hopkinson bar developed in the present investigation

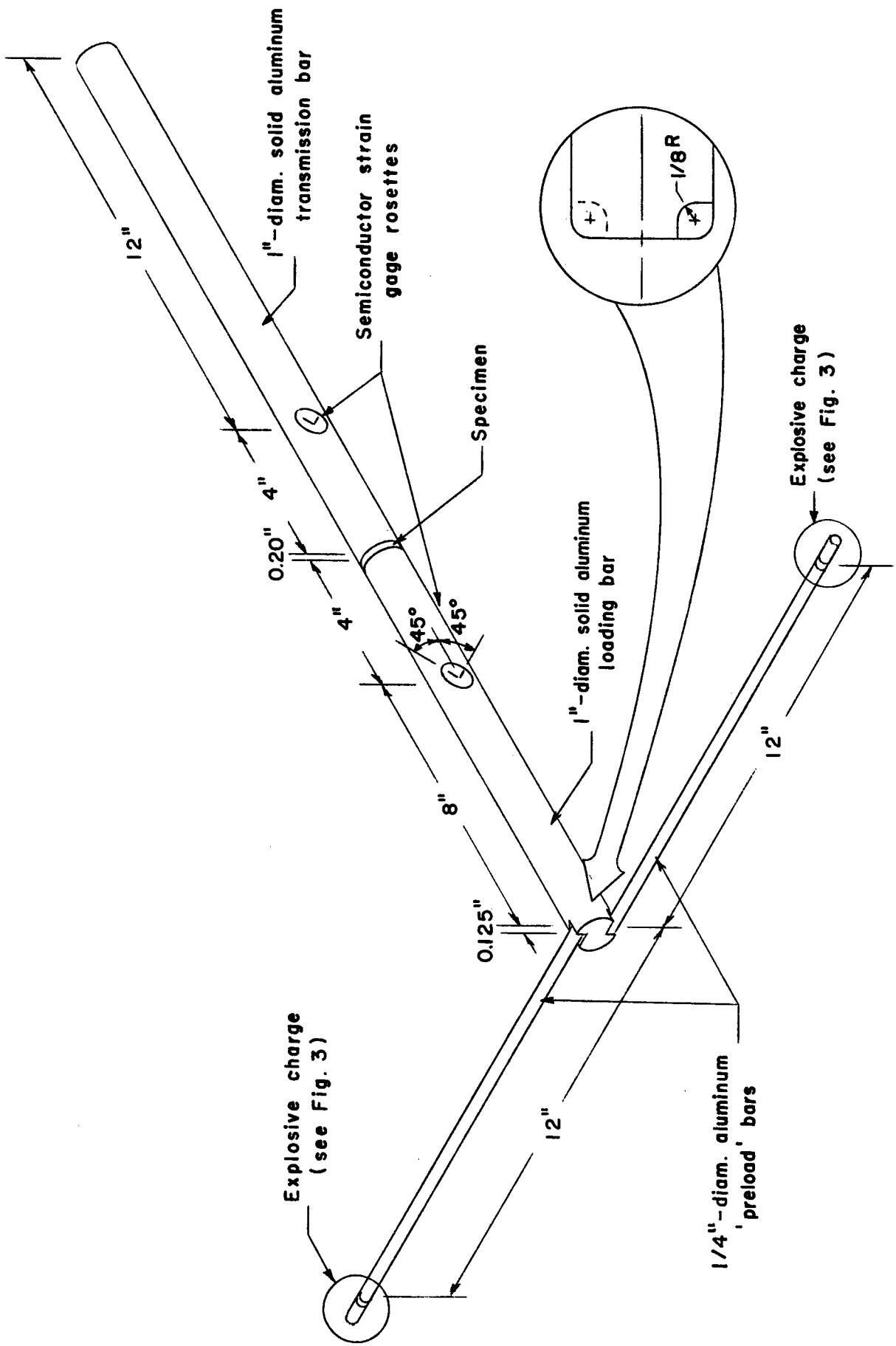


Fig. 2.—Sketch of the experimental setup

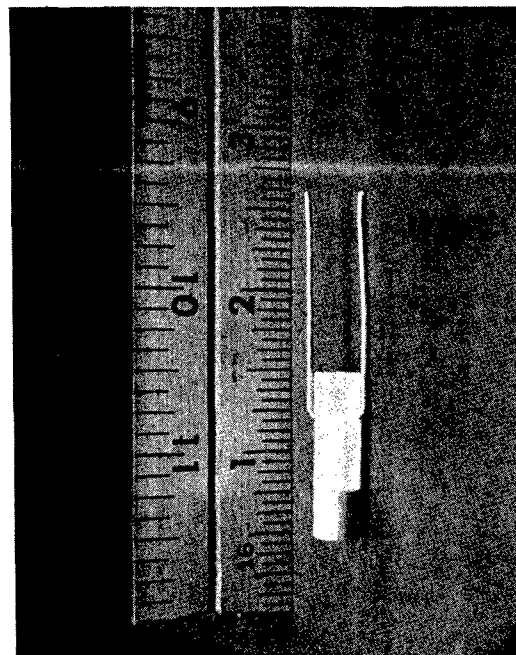
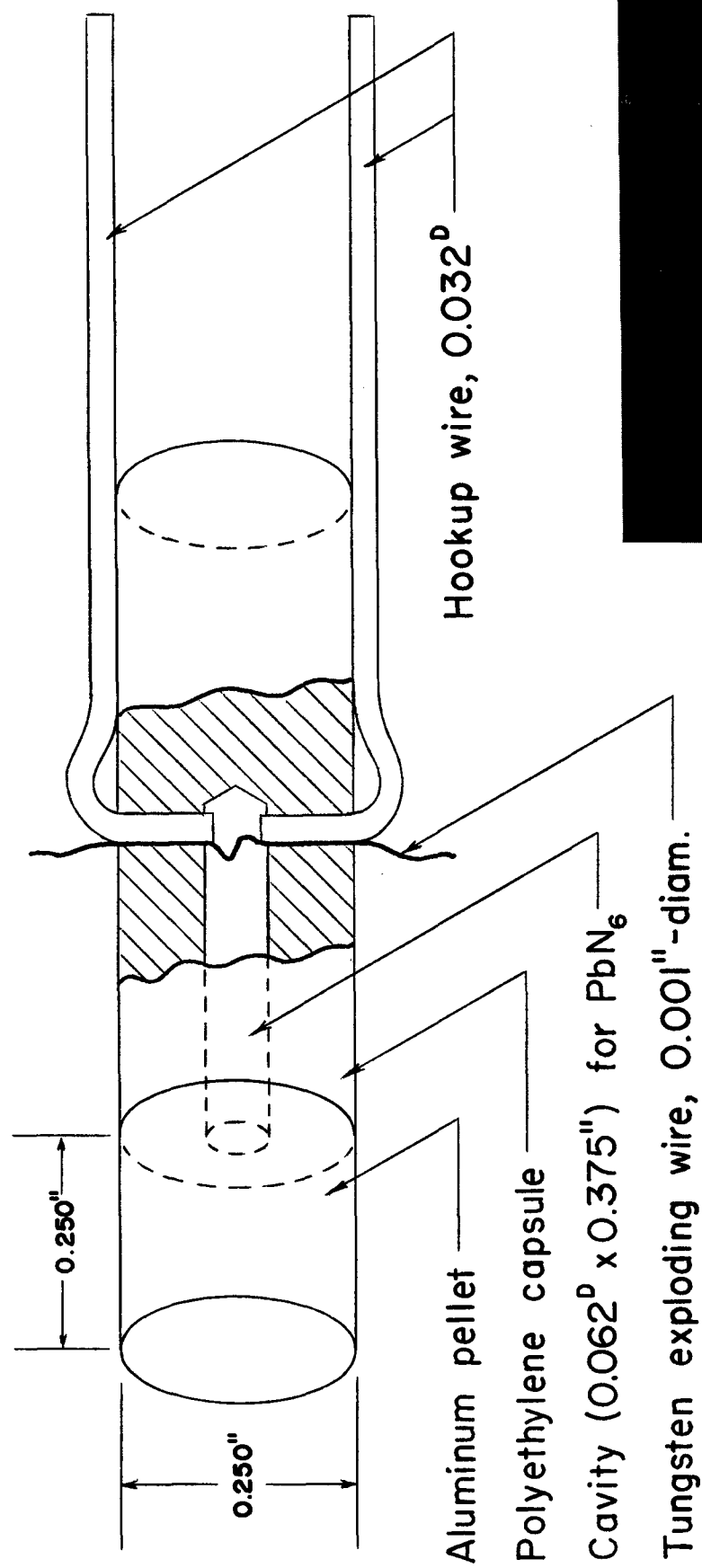


Fig. 3.—Detail of explosive  
(lead azide) charge

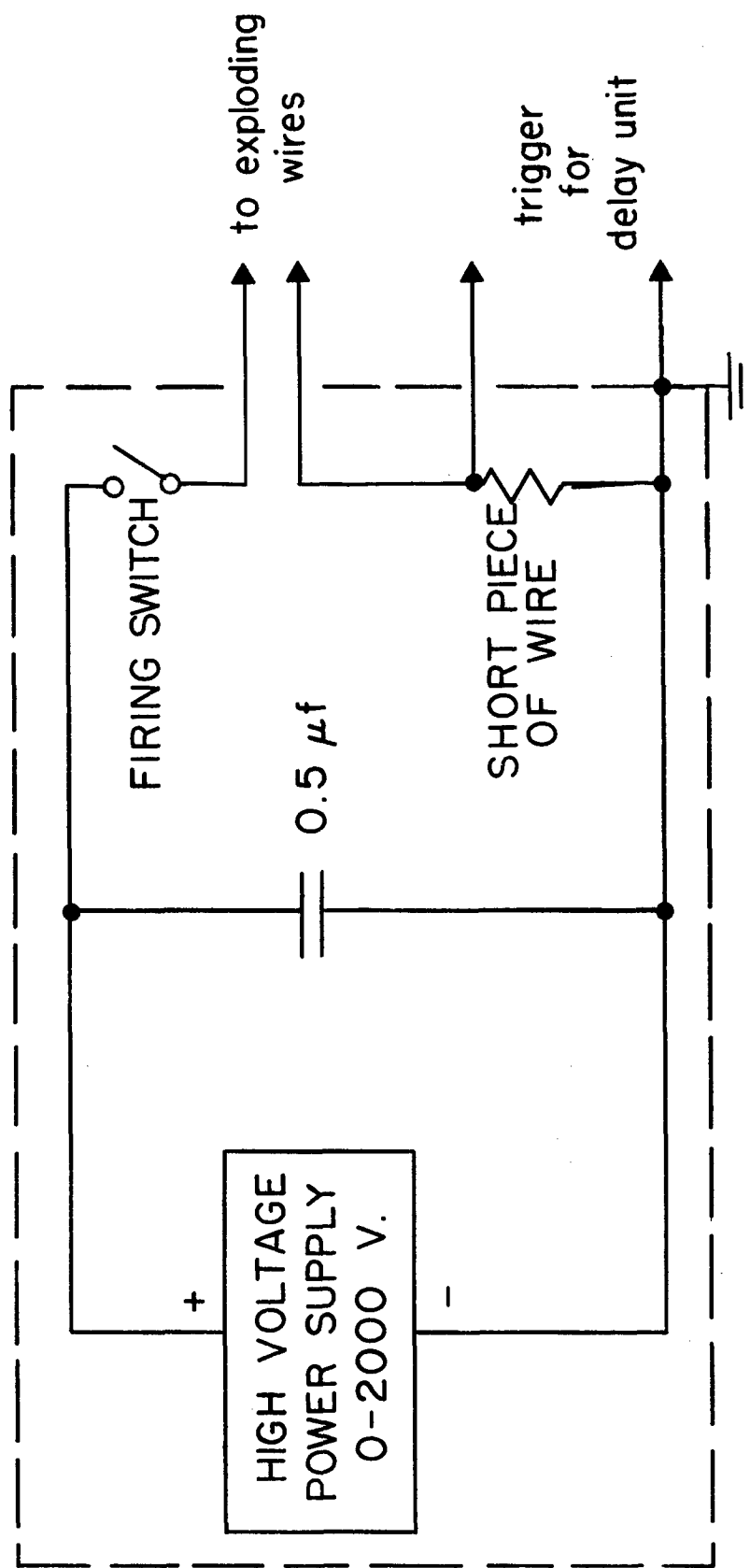


Fig. 4.—Schematic diagram of capacitor discharge system

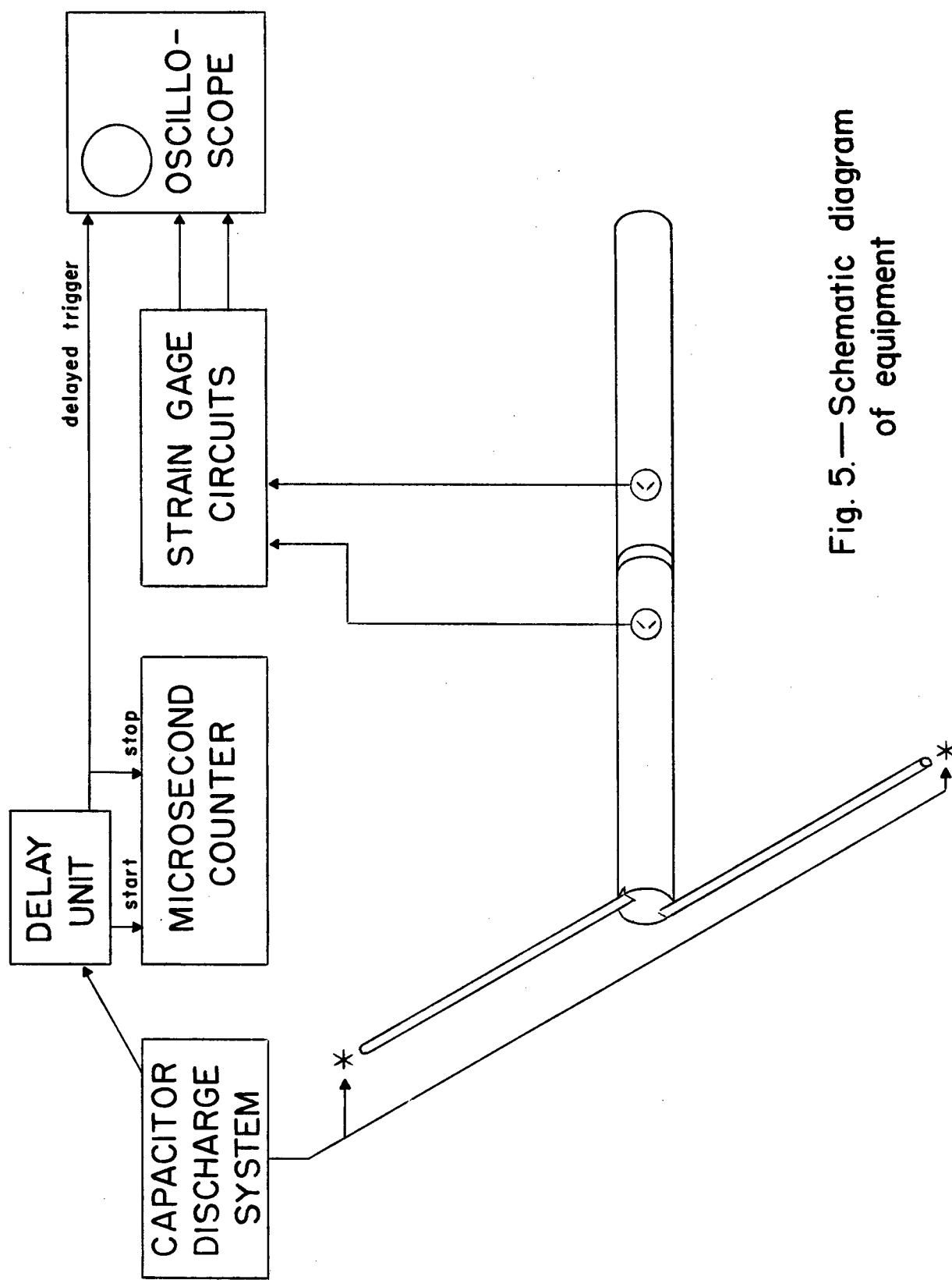


Fig. 5.—Schematic diagram of equipment

Fig. 6(a).—Longitudinal pulse in pre-load bar (top trace) and subsequent torsional pulse in main loading rod (lower trace) when one charge is used

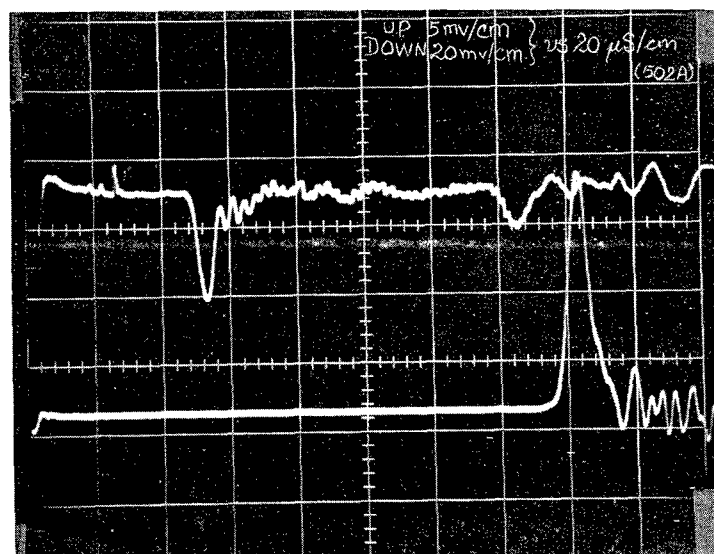


Fig. 6(b).—Torsional strain gage responses in loading bar (upper trace) and in transmission bar (lower trace) when dummy specimen and one charge are used

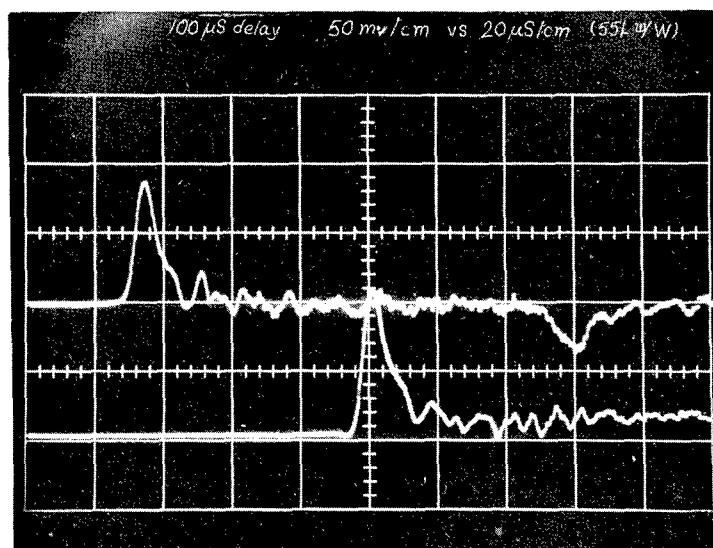


Fig. 6(c).—Same as in Fig. 6(b) except that two charges are used

